J B Academy, Ayodhya
Half Yearly Examination 2025-26
Class-XII, Sub- Mathematics

Time : 3 hrs. **M.M : 80**

General instructions:-

1) **This question paper contains 38 questions. All the questions are compulsory.**
2) **Question paper is divided into five sections – Section A, B, C, D and E.**
3) **Internal choices have been provided.**

Section - A (Each of 1 mark)

1) Let R be the relation on the set A = {1, 2, 3, 4} given by
R = { (1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)}. Then,
a) R is reflexive and symmetric but not transitive.
b) R is reflexive and transitive but not symmetric.
c) R is symmetric and transitive but not reflexive.
d) R is an equivalence relation.
2) Let A = {1, 2, 3}. Then, the number of equivalence relations containing (1, 2) is
a) 1 b) 2 c) 3 d) 4
3) If $\tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$ = α, then x^2 =
a) Sin 2α b) sin α c) cos 2α d) cos α
4) The number of solutions of the equation $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$ is
a) 2 b) 3 c) 1 d) none of these
5) If A = $\begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$ and A = A^T, then
a) x = 0, y = 5 b) x + y = 5 c) x = y d) none of these
6) If A is a square matrix such that $A^2 = A$, then $(I + A)^3$ - 7A is equal to
a) A b) I – A c) I d) 3A
7) The area of the triangle with the vertices (-3, 0), (3, 0) and (0, k) is 9 sq. units. The value of k will be
a) 9 b) 3 c) -9 d) 6
8) If A is a 3×3 matrix such that $|A|$ = 8, then $|3A|$ =
a) 8 b) 24 c) 72 d) 216
9) If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then x =
a) 3 b) ± 3 c) ± 6 d) 6
10) The function f (x) = $\begin{cases} \frac{\sin 3x}{x}, & x \neq 0 \\ \frac{k}{2}, & x = 0 \end{cases}$ is continuous at x = 0, then k =
a) 3 b) 6 c) 9 d) 12
11) If f (x) = x sin $\frac{1}{x}$, x ≠ 0, then the value of the function at x = 0, so that the function is continuous at x = 0, is
a) 0 b) -1 c) 1 d) indeterminate
12) The value of k which makes f (x) = $\begin{cases} \sin\frac{1}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ continuous at x = 0, is
a) 8 b) 1 c) – 1 d) none of these

13) Differential coefficient of sec ($\tan^{-1}$ x) is

a) $\frac{x}{1+x^2}$ b) x $\sqrt{1+x^2}$ c) $\frac{1}{\sqrt{1+x^2}}$ d) $\frac{x}{\sqrt{1+x^2}}$

14) If f (x) = $\tan^{-1}\sqrt{\frac{1+\sin x}{1-\sin x}}$, 0 ≤ x ≤ $\frac{\pi}{2}$, then f ' $\left(\frac{\pi}{6}\right)$ is

a) $-\frac{1}{4}$ b) $-\frac{1}{2}$ c) $\frac{1}{4}$ d) $\frac{1}{2}$

15) If y = $\left(1+\frac{1}{x}\right)^x$, then $\frac{dy}{dx}$ =

a) $\left(1+\frac{1}{x}\right)^x\left\{\log\left(1+\frac{1}{x}\right)-\frac{1}{x+1}\right\}$ b) $\left(1+\frac{1}{x}\right)^x$ log $\left(1+\frac{1}{x}\right)$

c) $\left(x+\frac{1}{x}\right)^x\left\{\log(x+1)-\frac{x}{x+1}\right\}$ d) none of these

16) If $x^y = e^{x-y}$, then $\frac{dy}{dx}$ is

a) $\frac{1+x}{1+\log x}$ b) $\frac{1-\log x}{1+\log x}$ c) not defined d) $\frac{\log x}{(1+\log x)^2}$

17) $\int \frac{x}{4+x^4}\, dx$ is equal to

a) $\frac{1}{4}$ $\tan^{-1} x^2$ + C b) $\frac{1}{4}$ $\tan^{-1}\left(\frac{x^2}{2}\right)$ c) $\frac{1}{2}$ $\tan^{-1}\left(\frac{x^2}{2}\right)$ d) none of these

18) $\int x \sec x^2\, dx$ is equal to

a) $\frac{1}{2}$ log (sec x^2 + tan x^2) + C b) $\frac{x^2}{2}$ log (sec x^2 + tan x^2) + C

c) 2 log (sec x^2 + tan x^2) + C d) none of these

19) $\int \frac{1}{1+\tan x}\, dx$ =

a) $\log_e$ (x + sin x) + C b) $\log_e$ (sin x + cos x) + C

c) 2 $\sec^2\frac{x}{2}$ + C d) $\frac{1}{2}${ x + log (sin x + cos x) } + C

20) The value of $\int \frac{1}{x+x\log x}\, dx$ is

a) 1 + log x b) x + log x c) x log (1 + log x) d) log (1 + log x)

Section - B (Each of 2 marks)

21) Let R be a relation on the set of all lines in a plane defined by (L_1, L_2) ∈ R given that L_1 is parallel to L_2. Show that R is an equivalence relation.

22) For the principle values, evaluate the following: $\tan^{-1}\sqrt{3}$ - $\sec^{-1}$ (-2).

23) Using determinants, find the area of the triangle whose vertices are (-2, 4), (2, -6) and (5, 4). Are the given points collinear?

24) Show that f(x) = $|x|$ is not differentiable at x = 0.

25) The radius of a circle is increasing uniformly at the rate of 4 cm/sec. Find the rate at which the area of the circle is increasing when the radius is 8 cm.

Section - C (each of 3 marks)

26) Find whether the following function is one – one or not:

F : R → R given by $f(x) = x^3 + 2$ for all $x \in R$.

27) Prove that: $\tan^{-1}\left\{\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right\} = \frac{\pi}{4} + \frac{1}{2}\cos^{-1} x^2$, $-1 < x < 1$.

28) The monthly incomes of Aryan and Babban are in the ratio 3 : 4 and their monthly expenditure are in the ratio 5 : 7. If each saves ₹ 15000 per month, find their monthly incomes using matrix method.

29) Differentiate the following function with respect to x:

$\text{Cos}^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, $0 < x < 1$.

30) Evaluate: $\int \frac{1}{\sin(x-a)\cos(x-b)}\, dx$.

31) Find the intervals in which the function $f(x) = 2x^3 - 9x^2 + 12x + 15$ is

i) increasing   ii) decreasing

Section - D (Each of 4 marks)

32) Three students, Neha, Rani, and Sam go to a market to purchase stationery items. Neha buys 4 pens, 3 notepads and 2 erasers and pays ₹ 60. Rani buys 2 pens, 4 notepads and 6 erasers for ₹ 90. Sam pays ₹ 70 for 6 pens, 2 notepads and 3 erasers.

Based on the above information, solve the following questions:

1) Form the equations required to solve the problem of finding the price of each item, and express it in the matrix form AX = B.
2) Find $|A|$ and confirm if it is possible to find A^{-1}.
3) Find A^{-1}, if possible and write the formula to find X.

   OR

   Find $A^2 - 8I$, where I is an identity matrix.

33) A technical company is designing a rectangular solar panel installation on a roof using 300 meters of boundary material. The design includes a partition running parallel to one of the sides dividing the area (roof) into two sections.

Let the length of the side perpendicular to the partition be x meters and with parallel to the partition be y meters.

Based on the above information, solve the following questions:

1) Write the equation for the total boundary material used in the boundary and parallel to the partition in terms of x and y.
2) Write the area of the solar panel as a function of x.

3) Find the critical points of the area function. Use second derivative test to determine critical points at the maximum area. Also, find the maximum area.

OR

Using first derivative test, calculate the maximum area the company can enclose with the 300 meters of boundary material, considering the parallel partition.

34) Let f(x) be a real valued function. Then:

Left Hand Derivative (LHD): Lf ' (a) = $\lim_{h\to 0} \frac{f(a-h)-f(a)}{-h}$,

Right Hand Derivative (RHD): Rf ' (a) = $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$,

Also, a function f(x) is said to be differentiable at x = a if its LHD = RHD at x = a exists and both are equal.

For the function f(x) = $\begin{cases} |x-3|, & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$.

Based on the given information, solve the following questions:

1) What is RHD of f(x) at x = 1?
2) What is LHD of f(x) at x = 1?
3) Check whether the function f(x) is differentiable at x = 1.

   OR

   Find f '(2) and f ' (-1).

Section - E (Each of 5 marks)

35) If $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix}$ A = $\begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$, find A.

OR

If A is a square matrix such that $A^2 = I$, then find the simplified value of $(A - I)^3 + (A + I)^3 - 7A$.

36) If A = $\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1} A^{-1}$.

37) Sand is pouring from a pipe at the rate of 12 cm^3/sec. The falling sand forms a cone on the ground in such a way that, the height of the cone is always 1/6th of the radius of the base. How fast is the height of the sand – cone increasing when the height is 4 cm?

OR

Find the intervals in which the function f(x) = log (1 + x) - $\frac{2x}{2+x}$ is increasing or decreasing (for every x > -1).

38) Evaluate: $\int \sin 2x \tan^{-1}(\sin x)\, dx$.

OR

Evaluate: $\int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}\, dx$.